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EQUILIBRIUM AND POWER BALANCE CONSTRAINTS ON A QUASI-STATIC, OHMICALLY-HEATED FRC

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I. Introduction

In present experiments, FRC's (field-reversed configurations) are generated on "dynamic" time scales using pulsed high-power theta-pinch technology,[1] which does not easily extrapolate to reactor-size devices. The attractiveness of FRC reactor scenarios would be enhanced by the development of quasi-static (i. e. formation time >> Alfven time) formation techniques requiring moderate power levels. In this report the quasi-static formation of FRC plasmas is analytically investigated.[2,3] The set of equations which yield the time evolution of the ohmically-heated-plasma parameters, under the constraints of radial equilibrium and plasma energy losses, are presented. Subject to the simplifying assumptions used in the model, this equation set is completely general and would apply to any ohmically-heated FRC. A sample calculation is presented in which the FRC azimuthal current, I_θ, is generated by the rotating-magnetic-field (RMF) technique.[4]

II. Governing Equations

An infinitely long, ohmically-heated FRC is considered. The electron and ion energy balance equations for a unit volume of the plasma are,

$$\frac{3}{2}\frac{d}{dt}(nkT_e) = \eta J_\theta^2 - P_{rad} - \frac{3}{2} nk\left[\frac{(T_e - T_i)}{\tau_{eq}} + \frac{T_e}{\tau_{Ee}}\right] \tag{1a}$$

$$\frac{3}{2}\frac{d}{dt}(nkT_i) = \frac{3}{2} nk\left[\frac{(T_e - T_i)}{\tau_{eq}} - \frac{T_i}{\tau_{Ei}}\right], \tag{1b}$$

where the resistivity, η, is assumed classical. $P_{rad} = n(t)\sum F_z n(0)L_z$, is the radiation power due to the initial fraction F_z of impurity element z with the cooling rate, L_z, as computed by Post, et. al.[5]. τ_{eq} is the electron-ion equilibration time. τ_{Ei} and τ_{Ee} are the ion and electron thermal energy confinement times respectively, with $\tau_{ej} = \xi_j \tau_{jj}(r_w/2\rho_j)^2$ where, $\xi_j = 1$ for classical conduction losses and $\xi_j < 1$ for losses greater than classical, τ_{jj} is the species self-collision time, r_w is the wall radius, and ρ_j is the gyroradius at the separatrix.

The radial distribution of particle density, n, and azimuthal current density, j_θ, are specified by assuming rigid-rotor profiles, $n = n_{max}\,\mathrm{sech}^2 K[(r/r_o)^2 - 1]$ where r_o is the magnetic axis, and $j_\theta = ne\omega r$. It is further assumed that the FRC is contained within a flux-conserving wall of radius r_w, so that $r_s \approx r_w$, where $r_s = \sqrt{2}r_o$ is the separatrix radius. Taking $K \approx 2$, conservation of particles yields $n_{max} \approx 2n_o$ where n_o is the initial fill density. The radial equilibrium constraint requires

$$n_{max}\,k(T_e + T_i) = B_w^2/2\mu_o, \tag{2}$$

where the magnetic field at the wall, B_w, is constrained through Ampere's law by the FRC azimuthal current,

$$B_w = \frac{\mu_o}{2}\int_0^{\rho} j_\theta\, dr$$

Ohmic dissipation results in FRC heating on relatively long time scales. This presents the possibility of "puff-gas injection" during the heating process. In anticipation of the example calculation given in section III, the fill density, n_o, is allowed to increase uniformly with temperature as,

$$n_o(t) = n_o(0)[(T_e + T_i)/T_e(0)]^{1/(2\gamma+1)} , \qquad (4)$$

where $\gamma > 0$, $T_e(0)$ and $n_o(0)$ are the electron temperature and density at time $t = 0$, and $T_i(0) \approx 0$. Temperature is assumed to be independent of radius. The equations which result from integrating Eqs. 1a and 1b over the plasma volume, using the above expressions, can be solved to yield the time evolution of the plasma parameters for a given r_w, γ, and the initial equilibrium conditions, $T_e(0)$, $n_o(0)$ and $j_\theta(0)$.

III. Example Numerical Calculation ; FRC's Formed by the RMF Technique

Figure 1 illustrates the RMF generation of an FRC. This technique has been successfully demonstrated in small-scale devices.[4,6,7] According to Blevin and Thonemann,[4] the electrons are tied to the rotating field lines, of magnitude B_o, resulting in a rigid-rotor current distribution $j_\theta = ne\omega r$ when, $\omega_{ce} > \omega > \omega_{ci}$ and $\nu_{ei}/\omega_{ce} < 1$, where ω is the rotating-field frequency, ω_{ce} and ω_{ci} are the electron and ion cyclotron frequencies (with respect to B_o) and ν_{ei} is the electron-ion collision frequency.

Integrating j_θ over radius yields, $I_\theta \propto \omega n_o$ for the total equilibrium current. This current can be maintained by programming in time the rotating-field frequency, ω, and/or n_o through puff-gas injection. Allowing $\omega \propto n_o^\gamma$, the radial equilibrium constraints require that $n_o(t)$ vary with temperature as shown in Eq. 4 and so

$$\omega = \omega(0)[(T_e + T_i)/T_e(0)]^{\gamma/2(\gamma+1)} \qquad (5)$$

where $\omega(0)$ is the value of ω at $t = 0$. Note that as $\gamma \to \infty$, $n_o(t) \to n_o(0)$ (i.e., no gas injection) and $\omega \propto (T_e + T_i)^{1/2}$.

The numerical results of Hugrass and Grimm,[8] show that the rotating field can penetrate and be sustained within the plasma if the penetration condition is satisfied, $\nu_{ei} r_w/\omega_{ce}\delta < 1$, where δ is the classical skin depth. Taking, from past experimental results,[6] $\omega = 5\omega_{ci}$ which sets an upper bound on B_o, and using Eqs. 4 and 5 it can be shown[2] that the penetration condition has the functional form,

$$\frac{\nu_{ei}}{\omega_{ce}} \frac{r_w}{\delta} = F[n_o(0),\ \omega(0),\ T_e(0),\ r_w,\ (1 + T_i/T_e)/(T_e/T_e(0))^\lambda \qquad (6)$$

where $\lambda = [(\gamma/2 - 1)/(2\gamma + 1)] + 3/4$. If $F < 1$ at $t = 0$, then for $\gamma > 1/8$ the penetration condition is satisfied for all time t. Thus, the bounds on γ are $1/8 \le \gamma < \infty$. The initial rotating field frequency, $\omega(0)$, is obtained from the combination of the Eqs. 2 and 3 evaluated at $t = 0$, and can be written as,

$$\omega(0) = C_1\ [T_e(0)/n_o(0)]^{1/2}/r_w^2, \qquad (7)$$

where C_1 is a constant. Subatuting Eq. 7 into Eq. 6, evaluated at $t = 0$, yields,[2] for the initial fill density,

$$n_o(0) = C_2\ [T_e(0)/r_w^2]^{4/5}, \qquad (8)$$

where C_2 is a constant.

For the initial conditions $T_e(0) = 2$ eV, $T_i(0) \approx 0$ and $r_{w3} = 40$ cm, Eqs. 7 and 8 give $\omega(0) = 1.3 \times 10^5$ rad/sec and $n_o(0) = 1.5 \times 10^{12}$ cm^{-3} respectively. The solution of Eqs. 1a and 1b for $\gamma = \infty$ (constant fill density) and $\gamma = 1/8$ (maximum rate of gas injection), for the above set of conditions, is plotted in Fig. 2. Radiation and transport losses have been neglected so these curves give the upper bounds on T_e and T_i, assuming classical resistivity. The corresponding $n_o(t)$ and ω (t) for this case are shown in Fig. 3. Although gas injection results in a somewhat lower T_e, it is technologically advantageous in view of the significantly smaller increase in rotating field frequency, ω, than required without gas injection. The effect of impurity radiation is shown in Fig. 4, where T_e is given for $\gamma = 1/8$ and oxygen impurity fractions, ranging from 0 to 15% of the initial fill density. The relatively weak effect of radiation on the temperature time history is attributable to low initial density $n_o(0)$. The scaling of T_e with wall radius is displayed in Fig. 5 for various times. As can be seen, the time required to ohmicaly heat quasi-statically formed FRC's to temperatures of fusion interest increases with the device radius squared. Figures 6 and 7 show the effects of cross-field thermal conduction on the electron temperature time-history for $r_w = 40$ cm, $\gamma = 1/8$, and 2% oxygen impurity. Electron thermal conduction losses many times faster than classical can be tolerated (Fig. 6.). However, ion thermal transport (Fig. 7.) only about a factor of two greater than classical is sufficient to clamp T_e at uninteresting values; a 1-D model is required to adequately investigate this effect. For the $\gamma = \infty$ (constant density) case, ion energy transport is unimportant since the low initial density results in a characteristic electron-ion equilibration time that greatly exceeds the ohmic heating time (see Fig. 2).

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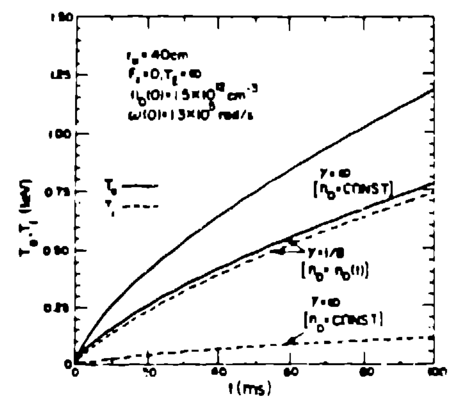
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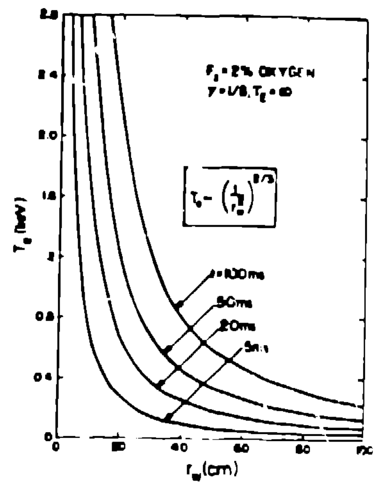
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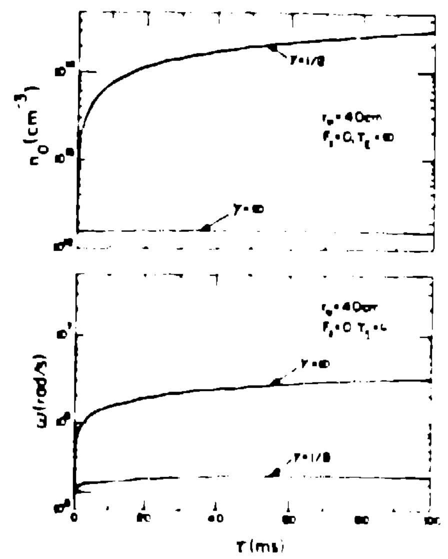
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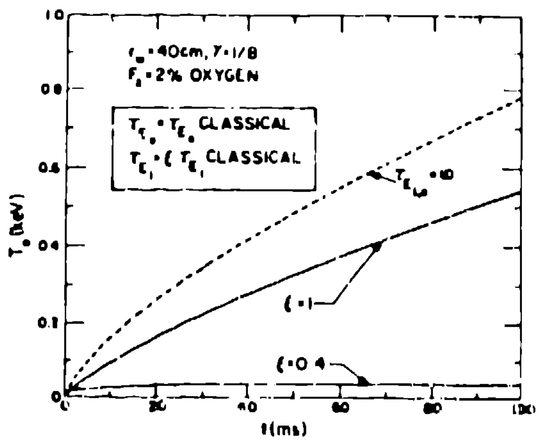
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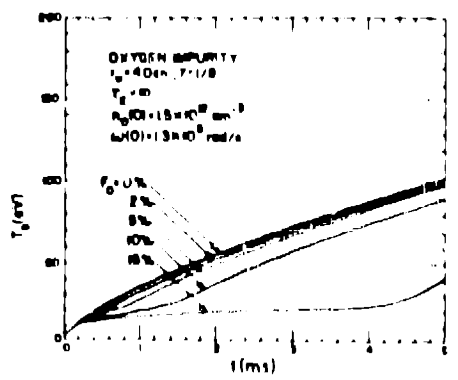
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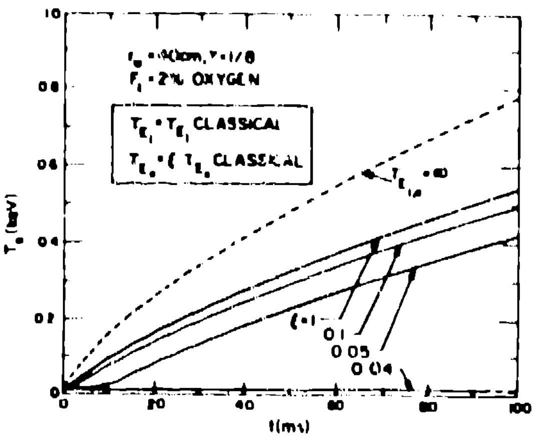
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